STAT 2593

Lecture 013 - The Binomial Distribution

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The Binomial Distribution

Learning Objectives

1. Understand the binomial distribution, its use cases, and its properties.

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  - We have $E[X] = np$, $\text{var}(X) = np(1-p)$, and

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

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- If you are sampling **without replacement** this is no longer a binomial distribution.
    - If the population is *very* large, then it will behave as an approximately binomial distribution.

Summary

- The binomial distribution characterizes the number of successes on repeated bernoulli trials.
- The binomial distribution relies on the assumptions of fixed number of trials, constant probability of success, and independence.
- There is a closed for PMF, expectation, and variance.